KENDRIYA VIDYALAYA CHITTARANJAN

HOLIDAY HOMEWORK MATHS XIIA (2023-24)

Q1. Prove that the relation R on the set $\mathbb{N} \times \mathbb{N}$ defined by
$(a , b) R (c , d) \Leftrightarrow a + d = b + c$ for all $(a , b) ; (c , d) \in \mathbb{N} \times \mathbb{N}$
is an equivalance relation .

Also find the equivalance classes [(2,3) and (1,3)]

Q2. Let $\mathbb{N}$ denote the set of all natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a ,b) R (c ,d) \Leftrightarrow ad(b+c) = b\ c\ (a +d)$

Check whether R is a equivalance relation on $\mathbb{N} \times \mathbb{N}$

Q3. Check whether the relation R on $\mathbb{R}$ defined by $R = \{ (a ,b): a \leq b^3\}$is reflexive, symmetric or transitive

Q.4 Given a non empty set **X** , consider P(X) which is the set of all possible subset of X. Define a relation in P(X) as follows:

For subsets A,B in P(X) , ARB if $A \subset B$. Is R an equivalance relation on P(X) ?

Justify your answer .

Q.5 Give an example of a relation, which is

(i) Symmetric but neither reflexive nor transitive .

(ii) Transitive but neither reflexive nor symmetry

(iii) Reflexive & symmetry but not transitive

(iv) Reflexive & Transitive but not symmetric

(v) Symmetric & transitive but not reflexive

Q.6 Show that the function f: $\mathbb{R} \rightarrow \mathbb{R}$ given by $f(x)=\cos x$ for all $x \in \mathbb{R}$ is neither one- one nor onto

Q7. Show that the function f: $\mathbb{R} \rightarrow \{x \in \mathbb{R} : -1<x<1\}$ defined by

$f(x)=\frac{x}{1+|x|}$, $x \in \mathbb{R}$ is one-one onto function .

Q.8 Show that the function f: $\mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3 + x$ is a bijection

Q.9 Prove that the greatest integer function given by f(x) = [x] is neither one-one nor onto . [x]=greatest integer less than or equal to x

Q.10 Show that the signum function f: $\mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} 1 \; if \; x > 0 \\ 0 \; if \; x = 0 \\ -1 \; if \; x < 0 \end{cases}$$ is neither one-one nor onto

Q11. Let A & B be two sets . Show that f: A X B→ B X A such that f(a ,b)=(b ,a) is bijective function.

Q.12 Prove that the followings:

(i) $\sin^{-1}\frac{1}{x} = \csc^{-1} x$
(ii) $\sin^{-1}(-x) = -\sin^{-1} x$
(iii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$
(iv) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

Q.12 Find the domain of f(x)= $\cos^{-1} 2x + \sin^{-1} x$

Q.13 Find the principal value of $\cos^{-1}(-1/2)$

Q.14 Find A^2-5A+6I=0, If $A=\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Q.15 A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of: (a) 1800